Spring 2007 – Exam 2 – Answer Key

1. a. X = passenger demand (of a randomly selected flight)

   $X \sim N(\mu = 125, \ \sigma = 40)$

   b. $P(X \geq 100) = P\left(\frac{X-125}{40} \geq \frac{100-125}{40}\right) = P(Z \geq -0.63) = 0.5 + 0.2357 = 0.7357$

   c. $P(75 < X < 200) = P\left(\frac{75-125}{40} < \frac{X-125}{40} < \frac{200-125}{40}\right) = P(-1.25 < Z < 1.88)$
   $= 0.3944 + 0.4699 = 0.8643$

   d. $P(X > 182) = P\left(\frac{X-125}{40} > \frac{182-125}{40}\right) = P(Z > 1.43) = 0.5 - 0.4236 = 0.0764$

   e. $P(X > h) = 0.04 \rightarrow P\left(\frac{X-125}{40} > \frac{h-125}{40}\right) = 0.04 \rightarrow P(Z>z) = 0.04$

   z = 1.75

   $1.75 = \left(\frac{h-125}{40}\right) \rightarrow h = (1.75)(40) + 125 =$ **<u>195</u>**

2. a. X = hair growth length per year $X \sim U(a = 4, b = 8)$

   b. $P(3 \leq X \leq 5.7) = P(4 \leq X \leq 5.7) = \left(\frac{1}{8-4}\right)(5.7 - 4) = (0.25)(1.7) = 0.425$

   c. $E[X] = \mu = \mu_x = (a+b)/2 = (4+8)/2 = 6$

   d. $P(X \geq 6.5) = P(X > 6.5) = \left(\frac{1}{8-4}\right)(8 - 6.5) = (0.25)(1.5) = 0.375$

   e. $P(X \leq k) = 0.80$ **<u>OR</u>** $P(X < k) = 0.80$

   $(k - 4)\left(\frac{1}{8-4}\right) = 0.80$

   $k = (0.80)(4) + 4 =$ **<u>7.2</u>**

   <u>OR</u>

   $P(X \geq k) = 0.20$ **<u>OR</u>** $P(X > k) = 0.20$

   $(8 - k)\left(\frac{1}{8-4}\right) = 0.20$

   $k = 8 - (0.20)(4) =$ **<u>7.2</u>**

3. a. $\overline{X}$ = <u>sample</u> average score (of 121 randomly selected individuals)
   By C.L.T., $\overline{X} \sim N\left(\mu_{\overline{x}} = \mu = ?, \ \sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}} = \frac{4}{\sqrt{121}} = 0.3636\right)$

   b. $\overline{x} \pm z_{\alpha/2}\sigma/\sqrt{n} = 81 \pm (1.96)(0.3636) = 81 \pm 0.7127 = (80.2873, 81.7127)$

   c. We are 95% confident that the <u>population</u> average retention score is between 80.2873 and 81.7127 points.

4. a. X = <u>number of</u> people who prefer to eat the ears of a chocolate bunny first

   X~B(n = 15, p = 0.75)

   b. $P(X > 9) = P(X \geq 10) = 1 - P(X \leq 9) = 1 - 0.148 = 0.852$

   c. $P(6 < X \leq 14) = P(7 \leq X \leq 14) = P(X \leq 14) - P(X \leq 6) = 0.987 - 0.004 = 0.983$

   d. $P(X < 7.5) = P(X \leq 7) = 0.017$

5. a. $\overline{X}$ = <u>sample</u> mean age (of 35 randomly selected BBC members)

   By C.L.T., $\overline{X} \sim N\left(\mu_{\overline{x}} = 72,\ \sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}} = \frac{6}{\sqrt{35}} = 1.0142\right)$

   b. $P(\overline{X} \leq 69) = P(\overline{X} < 69) = P\left(\frac{\overline{X} - 72}{1.0142} \leq \frac{69 - 72}{1.0142}\right) = P(Z \leq -2.96) = 0.5 - 0.4985 = 0.0015$

   c. $P(\overline{X} = 72) = 0$

   d. X = age (of one randomly selected BBC member) $X \sim N(\mu = 72,\ \sigma = 6)$

   $$P(65 \leq X \leq 71) = P(65 < X < 71) = P\left(\frac{65 - 72}{6} < \frac{X - 72}{6} < \frac{71 - 72}{6}\right)$$

   $$= P(-1.17 < Z < -0.17) = 0.3790 - 0.0675 = 0.3115$$

6. a. X = <u>number of</u> first year students that planned to be employed between 10 and 19 hours per week

   X~B(n = 10, p = 0.43)

   b. $P(X=7) = C_7^{10}(0.43)^7(0.57)^3 = (120)(0.0027)(0.1852) = 0.0600$

   c. $P(6 < X < 9) = P(7 \leq X \leq 8) = P(X=7) + P(X=8)$

   $= 0.0600 + C_8^{10}(0.43)^8(0.57)^2$
   $= 0.0600 + (45)(0.0012)(0.3249)$
   $= 0.0600 + 0.0175 =$ **<u>0.0775</u>**

   d. $V[X] = \sigma^2 = \sigma_x^2 = np(1-p) = (10)(0.43)(0.57) = 2.451$

   $\sigma = \sigma_x = \sqrt{2.451} =$ **<u>1.5656</u>**

7. d. none of the above   8. c. cluster sampling

9. c. In repeated sampling, 95% of the intervals constructed would contain the population mean.

10. a. FALSE   b. TRUE   11. d. none of the above